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ON THE EFFECT OF THE EARTH'S ROTATION ON BODIES MOVING ON ITS SURFACE.

BY THE EDITOR.

THAT any meridian plane of a revolving sphere will rotate, with respect to a fixed plane, around a normal axis, which pierces the sphere at any point of its surface, with a uniform angular velocity equal to the angular velocity of the revolving sphere multiplied by the sine of the latitude of the point at which the normal axis pierces the surface of the sphere, is a geometrical truth which is wholly independent of any consideration of force whatever. (For a demonstration of this proposition see American Journal of Science for 1852, Vol. XIII, p. 212.)

As a consequence of the inertia of matter, a mass in motion, constrained to move about a center, by a constant force directed to that center, will develop a centrifugal force which will be directly proportional to the square of the velocity and inversely proportional to the radius of the circle.

If we put R for the earth's equatorial radius and V for its equatorial velocity per hour (sidereal time), we have for its equatorial centrifugal force

$$\frac{V^2}{R} = \frac{g}{289}; \; V = \frac{2\pi R}{24}. \tag{1}$$

If a velocity v be impressed upon a body, at any point P in latitude λ of the earth's surface, the body will, under the impulse of the impressed force and the earth's gravitating force, move in a fixed plane passing through the center of the earth and therefore intersecting its surface in a great circle of which the point P will be the extremity of a diameter.

Now, in accordance with the above proposition, a meridian plane will, in consequence of the earth's rotation, revolve, with respect to the fixed plane through P, about a normal axis at P, with a uniform angular velocity.

Hence, instead of supposing the moving body to remain in the fixed plane, if we suppose it to be deflected from its path by the rotation of a meridian plane around the normal at P, it will be, at every point of its path, forced to move with a uniform velocity at right angles to a tangent at that point, and hence will be compelled to describe a circle, and will, by the above proposition, return to P in the time occupied by the meridian plane in making one rotation about the normal at P, viz., 24 hours divided by sin λ; therefore the circumference of the circle in which the body is forced to move is $24v \div \sin\lambda$, and its radius is $24v \div 2\pi \sin\lambda$, and because the deflecting or centripetal force f acting on the body is to the earth's equatorial centrifugal force directly as the squares of their velocities and inversely as their radii;

$$\therefore f : \tfrac{1}{289}g :: \frac{v^2}{24v \div 2\pi \sin \lambda} : \frac{V^2}{R};$$

$$\text{or } f : \tfrac{1}{289}g :: \frac{2\pi v \sin \lambda}{24} : \frac{4\pi^2 R}{(24)^2}; \text{ whence}$$

$$f = \frac{\tfrac{1}{289}g\, 24v \sin \lambda}{2\pi R}. \tag{2}$$

Equation (2) represents the total deflecting force at P which results from the rotation of the tangent plane around the normal, and is entirely independent of the direction in which the body moves, provided the centrifugal force of the moving body corresponds with that of the earth's surface at the point P; but this is only the case when the initial motion of the body is in the meridian of P.

Let β denote the angle between the meridian plane and the initial direction in which the body moves, estimated from the south toward the west, then is the centrifugal force of the earth's surface at P to the centrifugal force in the same orbit due the velocity v as $V^2 \cos^2\lambda$ is to $v^2 \sin^2 \beta$, or

$$\tfrac{1}{289}g \cos \lambda : F :: V^2 \cos^2\lambda : v^2 \sin^2\beta,$$

from which we find

$$F = \frac{\tfrac{1}{289}g\, v^2 \sin^2 \beta \cos \lambda}{V^2 \cos^2 \lambda} = \frac{\tfrac{1}{289}g\, v^2 \sin^2 \beta}{V^2 \cos \lambda}.$$

Resolving F into horizontal and vertical components at P, we find for its horizontal component

$$F' = \frac{\tfrac{1}{289}g\, (24)^2 v^2 \sin \lambda \sin^2 \beta}{4\pi R^2 \cos \lambda} = \frac{24v \sin^2 \beta}{2\pi R \cos \lambda} \times \frac{\tfrac{1}{289}g\, 24v \sin \lambda}{2\pi R}. \tag{3}$$

Adding equations (2) and (3) we get for the total deflecting force at P

$$f + F' = \left(1 + \frac{24v \sin^2 \beta}{2\pi R \cos \lambda}\right) \times \frac{\tfrac{1}{289}g\, 24v \sin \lambda}{2\pi R}. \tag{4}$$

If we suppose the point P to be in latitude 87° 45′ and v to represent a westward velocity of 40 miles per hour, the body, though moving at the rate of 40 miles per hour relative to the earth's surface, will be in a state of absolute rest, and will therefore develop no centrifugal force about the earth's axis, and consequently it will tend to a state of equilibrium by moving toward the pole, or to the right, with a force repesented by the earth's centrifugal force at that latitude resolved in the direction of the tangent plane, i. e., with a force $= \tfrac{1}{289}g \cos \lambda \sin \lambda$; this result is obtained independently of any consideration of the rotation of the meridian about a normal axis at P, and from which there results precisely the same deflecting force at that point, as shown by equation (2). Hence equation (4) is true when $v = V \cos \lambda$, and is therefore true for all values of v and β.

If the moving body be a pendulum suspended at P in latitude λ, its centrifugal force about the earth's axis will correspond with that of the point P, and v in equation (3) will become zero, so that in whatever direction a pendulum may be vibrated, its deflecting force will be completely represented by f in equation (2); for the centrifugal force produced by the motion of the pendulum about its point of suspension is always in the plane of that motion and therefore has no tendency to deflect the pendulum out of that plane; but the velocity v' with which the pendulum passes the point P, if referred to the earth's axis as its center of motion, may be substituted for v in equation (2) and hence f in equation (2) represents the whole of the deflecting force, or reaction, which a pendulum vibrating with a maximum velocity v' would exert on the rotating meridian through P.

As the term F', equation (3), results from the difference between the centrifugal force of the moving body and that of the point P, about the earth's axis, if the moving body be a projectile this term will vanish, but in its stead we shall then have the term

$$F'' = \frac{v \cos \lambda \cos \beta}{\sin \lambda} \times \frac{\frac{1}{289} g \, 24 v \sin \lambda}{2 \pi R}, \tag{5}$$

so that, at the end of any time t, the deflecting force will be

$$f + F'' = \left(1 + \frac{v \cos \lambda \cos \beta}{\sin \lambda}\right) \times \frac{\frac{1}{289} g \, 24 v \sin \lambda}{2 \pi R}. \tag{6}$$

Because the second term in the parentheses in equation (4) vanishes when $\sin \beta = 0$ and the corresponding term in equation (6) vanishes when $\cos \beta = 0$, it follows that, for a projectile, the deflecting force is a maximum when the direction of the motion is in the plane of the meridian, while for a body moving on the earth's surface, as a locomotive on a straight track, the deflecting force is a maximum when the track is at right angles with the meridian through the origin of the motion.

In a paper on this subject published in Van Nostrand's *Engineering Magazine* for June, 1883, p. 468, Mr. Randolph says truly that, "While the earth was in a sufficiently fluid state its materials adjusted themselves into the form of an oblate spheroid . . . which is exactly that necessary to counteract the tendency of a detached mass to move on such surface by virtue of the centrifugal force"; but he fails to note that the centrifugal force we have to deal with is not $V \cos \lambda$, to which the earth's figure is adapted, but $V^2 \cos^2 \lambda \pm v^2 \sin^2 \beta$, the substitution of which introduces the term F' as above.